Dust charge fluctuations in a magnetized dusty plasma

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Charging currents of electrons and ions to a spherical dust grain in a uniform magnetized dusty plasma have been examined. It is found that the external magnetic field reduces the charging currents, thereby decreasing the dust charge fluctuation damping of a low-frequency electrostatic wave in a dusty plasma.

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It is well known [1,2] that dust charge fluctuations induce a damping of the dust-acoustic [3] and dust-ion-acoustic [4] waves in an unmagnetized dusty plasma. There has been a significant progress in the study of unmagnetized dusty plasma physics, which is summarized by Mendis [5]. However, space and laboratory dusty plasmas are usually confined in external magnetic fields. To the best of our knowledge, there does not exist a theory for the dust charge fluctuations and the consequent effect of the dust charge fluctuation damping of the low-frequency waves in the presence of an external magnetic field in a dusty plasma.

The charging of a spherical probe in a magnetic field has been investigated in connection with the charging of satellites and rockets in Earth's ionosphere and magnetosphere [6–8], by assuming that the electron Debye radius λ_{De} is usually much larger than the probe size a ($a \ll \lambda_{De}$) in space conditions. In the collisionless limit, strong magnetic field effect has been considered in the limits $\lambda_{mfp} \gg a$, and $\rho_e \ll a$, where λ_{mfp}, a and ρ_e are the mean-free path, the probe radius, and the electron gyroradius [9,10], respectively.

In general, the presence of an external magnetic field makes a dusty plasma anisotropic. Hence, charging currents to a spherical dust grain is different in different directions. However, in the presence of a very strong magnetic field, the orbits of the magnetized plasma particles are confined to one dimension along the field lines, as if they are "glued" to the magnetic field lines. Hence, the perturbed field component transverse to the magnetic field does not come into play, and the problem of charging currents becomes independent of the magnetic field.

In the present Brief Report, we consider the plasma regimes that are characterized by (i) $\lambda_{mfp} \gg a$, (ii) $a < \lambda_D$, (iii) $\rho_{e,i} \gg a$, and (iv) $\lambda_D \leq r_0$, where λ_D is the effective dusty plasma Debye radius [13] and r_0 is the intergrain distance. The first condition is valid in the collisionless limit, the second condition is valid for the usual dusty plasma parameters, the third condition is valid for a moderate magnitude of the magnetic field, and finally the fourth condition is true for the fact that the charging of the dust grain is due to both the electron and ion currents. These limits are usually valid in the context of charging dust grains in a laboratory dusty magnetoplasma.

We consider a dusty plasma in the presence of an external magnetic field $\hat{\mathbf{z}}B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the z axis and B_0 is the magnitude of the external magnetic field. The plasma constituents are electrons, ions, and negatively charged micron-sized spherical dust grains. Following Tsytovich *et al.* [11], we first discuss the effects of an external magnetic field on equilibrium charging currents. In a weak magnetic field, the electron gyroradius (ρ_e) is much smaller than the dust size and the change of dust charges is relatively small. Here, electrons rapidly approach the dust grain surface along the external magnetic field direction, so that fast electrons charging a grain may have a Boltzmann distribution. Accordingly, the orbital motion limited (OLM) electron current remains in tact and we have [12,13]

$$I_{e0} = -n_{e0}A_e u_e \exp(eV_d/T_e), \tag{1}$$

where n_{e0} is the unperturbed electron number density, A_e is the average dust charging cross section, which is somewhat smaller than the geometrical cross section πa^2 of a dust particle, $u_e = \sqrt{8/\pi}v_{te}$, $v_{te} = (T_e/m_e)^{1/2}$ is the electron thermal speed, T_e is the electron temperature, m_e is the electron mass, e is the magnitude of the electron charge, and V_d $(=\phi_d - \phi_p \sim Z_d e/a)$ is the potential of the dust grain relative to the plasma potential. On the other hand, ions are attracted by dust and their effective cross section A_i is larger than the geometrical cross section. The OLM ion current then reads

$$I_{i0} = n_{i0}A_i u_i(1 + eV_d/K), \tag{2}$$

where n_{i0} is the unperturbed ion number density, $u_i = \sqrt{8/\pi}v_{ti}$, $v_{ti} = (T_i/m_i)^{1/2}$ is the ion thermal speed, T_i is the ion temperature, m_i is the ion mass, and $K = m_i v_i^2/2$ is the kinetic energy of ions before encountering the grain potential. The bracketed term is the focusing factor. At equilibrium, we have

$$n_{i0} = n_{e0} + Z_{d0}n_{d0}, \tag{3}$$

where Z_{d0} is the number of electrons residing on the dust grain surface, and n_{d0} is the unperturbed dust number density. It turns out that for weak magnetic fields the dust charge remains the same.

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However, when the strength of the magnetic field becomes larger than a critical value where the electron gyroradius is equal to the collection radius of electrons on dust grains, only fast magnetized electrons would be involved in the charging process, while a fraction of low energy electrons would be reflected backwards along the magnetic field direction. Hence, the charging cross section for electrons would be smaller, resulting in lowering the electron current by a factor of 4 compared to that in the absence of a magnetic field [11]. If the ion gyroradius is still much smaller than the ion dust attraction size, the ions will be attracted to the dust grain with approximately the same rate and their effective cross section A_i will be much larger than the geometrical cross section πa^2. The ion current on the grain will then remain the same as in an unmagnetized plasma. The reduction of the electron current means a decrease of dust charges. The radial electric field E_r of a dust grain in its vicinity would produce insignificant azimuthal $(E_r B_0/c)$ drift of electrons if a/ρ_e is larger than $Z_d e^2/aT_e$ [11].

For stronger magnetic fields [viz., $B_0 \geqslant (c/ea)\sqrt{m_i T_i}$] in a plasma, the ion gyroradius becomes smaller or comparable to the dust size. Here, both the electron and ion currents are modified due to the strong magnetization of the plasma particles. Tsytovich *et al.* [11] treated this problem numerically and reported the dependence of dust charge on the external magnetic field strength as well as on the parameter $\mu = \sqrt{m_i T_i/m_e T_e}$, where $m_i/m_e (T_i/T_e)$ is the ion to electron mass (temperature) ratio. They found that the dust charge in a strong magnetic field can be substantially larger (up to 12 times) than that in the absence of the magnetic field (or in a weak magnetic field).

Next, we consider electrostatic perturbations $(\omega,\mathbf{k})$ accounting for dust charge fluctuations. The charging equation for dust particles in a dusty plasma is

$$d_t Q_{d1} = I_{e1} + I_{i1}, \tag{4}$$

where Q_{d1} is the perturbation of the dust charge in the presence of the perturbed electron and ion currents $I_{e1,i1}$ associated with the perturbed plasma particle distribution functions in the electrostatic field. Here, ω and $\mathbf{k}$ are the frequency and the wave vector, respectively.

To calculate the perturbed currents of the magnetized electrons and ions, we assume $\rho_{e,i} \geqslant a$, where ρ_j is the Larmor radius of the species j (j equals e for electrons, i for ions and d for dust grains). We employ the guiding-center coordinates [14,15] and obtain the perturbed distribution function in the presence of the electrostatic potential $\Phi(\mathbf{x},t)$ as

$$\begin{aligned} f_{j1}(\mathbf{x},\mathbf{v},t) = & -\frac{q_j \Phi(\mathbf{x},t)}{T_j} \sum_l \sum_n J_l\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) J_n\left(\frac{k_\perp v_\perp}{\omega_{cj}}\right) \\ & \times \exp[i(n-l)(\theta-\delta)] \\ & \times \left(1 - \frac{\omega}{\omega - k_\parallel v_\parallel - n\omega_{cj}}\right) f_{j0}, \end{aligned} \tag{5}$$

where $\Phi(\mathbf{x},t) = \Phi_0 \exp[i(\mathbf{k}\cdot\mathbf{x} - \omega t)]$, Φ_0 being the amplitude of the electrostatic perturbation, θ is the angle between $v_\perp$ and the x axis, δ is the angle between $k_\perp$ and the x axis, J_l is the lth order Bessel function of the first kind, and $\|(\perp)$ denotes a quantity parallel (perpendicular) to $\hat{\mathbf{z}}$.

Thus, in the presence of a low-frequency electrostatic field, the charging current perturbations are

$$\begin{aligned} I_{j1}(\mathbf{x},t) \equiv & \oint \mathbf{J}_j \cdot d\mathbf{S} \\ & = 2\pi a^2 q_j \int (v_\perp \cos\theta + v_\perp \sin\theta + v_\parallel) f_{j1} d\mathbf{v}, \end{aligned} \tag{6}$$

where f_{j1} is given by Eq. (5). After a straightforward calculation, we then obtain from Eq. (6)

$$\begin{aligned} I_{j1}(\mathbf{x},t) = & -4\pi a^2 q_j n_{j0} \frac{q_j \Phi(\mathbf{x},t)}{T_j} \\ & \times \exp\left(-\frac{q_j \Phi_G}{T_j}\right)\left(\frac{T_j}{2\pi m_j}\right)^{1/2} F_j, \end{aligned} \tag{7}$$

where $F_j = F_{j1} + F_{j2} + F_{j3}$, with

$$F_{j1} = \sqrt{\frac{\pi}{2}} \sum_n \frac{n\omega_{cj}}{k_\perp v_{tj}} I_n \exp(-b_j)\left[1 + \frac{\omega}{\sqrt{2}k_\parallel v_{tj}} Z(\xi_n)\right], \tag{8}$$

$$\begin{aligned} F_{j2} = & -i\sqrt{\frac{\pi}{2}} \frac{k_\perp v_{tj}}{\omega_{cj}} \sum_n \left[1 + \frac{\omega}{\sqrt{2}k_\parallel v_{tj}} Z(\xi_n)\right] \frac{d}{db_j}[I_n \\ & \times \exp(-b_j)], \end{aligned} \tag{9}$$

$$F_{j3} = \sqrt{\frac{\pi}{2}} \frac{\omega}{k_\parallel v_{tj}} \sum_n I_n \exp(-b_j)[1 + \xi_n Z(\xi_n)], \tag{10}$$

and $\xi_n = (\omega - n\omega_{cj})/\sqrt{2}k_\parallel v_{tj}$.

For $\omega \ll \omega_{cj}$, $k_\perp v_{tj} \ll \omega_{cj}$, and $k_\parallel v_{tj} \ll |\omega|, |\omega - n\omega_{cj}|$, the modification factor F_j reduces to

$$F_j \simeq \sqrt{\frac{\pi}{2}} (1-b_j)\left[\frac{k_\perp v_{tj}}{\omega_{cj}}\left(\frac{\omega}{\omega_{cj}} - i\frac{k_\parallel^2 v_{tj}^2}{\omega^2}\right) - \frac{k_\parallel v_{tj}}{\omega}\right]. \tag{11}$$

When $k_\parallel/k_\perp \ll \omega^2/\omega_{cj}^2$ and if we neglect the small imaginary term in Eq. (9), the modification factor is found to be

$$F_j \simeq \sqrt{\frac{\pi}{2}} \frac{k_\perp v_{tj}}{\omega_{cj}} \frac{\omega}{\omega_{cj}} (1-b_j), \tag{12}$$

which is less than unity.

We note from Eq. (7) that for $b_j \ll 1$ and $k_\parallel v_{tj} \ll \omega$, the perturbed currents in a uniform magnetized dusty plasma is reduced by a factor F_j from the unperturbed currents to the grains.

For $|\omega| \ll \omega_{cj}, k_\perp v_{tj} \ll \omega_{cj}$, and $k_\parallel v_{tj} \gg |\omega|, |\omega - n\omega_{cj}|$, we obtain from Eqs. (8)–(10)

$$F_j \simeq \sqrt{\frac{\pi}{2}} (1-b_j)\left(\frac{\omega}{k_\parallel v_{tj}} + i\frac{k_\perp v_{tj}}{\omega_{cj}}\right). \tag{13}$$

For $|\omega|/k_\parallel v_{tj} \gg k_\perp v_{tj}/\omega_{cj}$ we can neglect the imaginary part in Eq. (13), and F_j again becomes less than unity. Equations (12) and (13) reveal that F_j is less than unity for both high- and low-parallel phase velocity waves.

Taking $n_j = n_{j0} + n_{j1}$ and $Q_d = Q_{d0} + Q_{d1}$, we obtain from Eq. (4)

$$Q_{d1} = -i\beta(\omega)\Phi/\omega, \tag{14}$$

where

$$\beta(\omega) = \frac{a^2}{\sqrt{2\pi}}\left[\frac{\omega_{pe}}{\lambda_{De}}\exp\left(\frac{e\Phi_G}{T_e}\right)F_e(\omega) + \frac{\omega_{pi}}{\lambda_{Di}}\left(1 - \frac{e\Phi_G}{T_i}\right)F_i(\omega)\right]. \tag{15}$$

Here, $\lambda_{Dj} = v_{tj}/\omega_{pj}$, $\omega_{pj} = (4\pi n_{j0} q_j^2/m_j)^{1/2}$, and $F_{e,i}(\omega)$ are given by Eq. (10) or Eq. (11), depending on the conditions of the wave perturbation and the plasma parameters.

To demonstrate how the external magnetic field affects the damping of a low-frequency electrostatic mode propagating nearly perpendicular to the magnetic field direction, we consider a magnetized dusty plasma with finite electron and ion temperatures. For $\omega_{cd}, kv_{td}, k_\parallel v_{ti}, k_\parallel v_{te} \ll |\omega| \ll \omega_{ci}, \omega_{ce}$, and $b_{e,i} \ll 1$, the electrons and ions are strongly magnetized, while cold dust grains are unmagnetized.

Poisson's equation in the presence of a low-frequency mode and dust charge fluctuations is

$$k^2\Phi + 4\pi(n_{e1}e - n_{i1}e - n_{d1}Q_{d0} - n_{d0}Q_{d1}) = 0, \tag{16}$$

where Q_{d1} is given by Eq. (16) for F_j given by Eq. (11). Inserting $n_{j1} = -(k^2\chi_j/4\pi q_j)\Phi$, where $\chi_e \simeq (\omega_{pe}/\omega_{ce})^2 \times (k_\perp/k)^2 - (\omega_{pe}/\omega)^2(k_\parallel/k)^2$, $\chi_i \simeq (\omega_{pi}/\omega_{ci})^2(k_\perp/k)^2 - (\omega_{pi}/\omega)^2(k_\parallel/k)^2$, and $\chi_d \simeq -(\omega_{pd}/\omega)^2$ in Eq. (17), we obtain $\epsilon(\omega,\mathbf{k})\Phi = 0$, where the dielectric constant of a magnetized dusty plasma is

$$\epsilon(\omega,\mathbf{k}) = 1 + \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1 + \frac{n_{e0}m_e}{n_{i0}m_i}\right) - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2}\left(1 + \frac{n_{i0}m_e}{n_{e0}m_i}\right) - \frac{\omega_{pd}^2}{\omega^2} + i\frac{4\pi n_{d0}\beta(\omega)}{k^2\omega}. \tag{17}$$

Neglecting the dust charge fluctuations, one readily obtains from $\epsilon(\omega,\mathbf{k}) = 0$ the dust-lower-hybrid (DLH) wave frequency [16]

$$\omega = \frac{k}{k_\perp}\frac{\omega_{pd}\omega_{ci}}{\omega_{pi}}\left(1 + \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega_{pd}^2}\right)^{1/2} \equiv \omega_r, \tag{18}$$

where $n_{i0}m_e \ll n_{e0}m_i$ has been assumed. However, dust charge fluctuations introduce a damping of the DLH waves with a damping rate

$$\gamma_L = -4\pi n_{d0}\beta(\omega_r)/k^2, \tag{19}$$

which is obtained from Eq. (17) by letting $\omega = \omega_r + i\gamma_L$, with $\gamma_L^2 \ll \omega_r^2$. Here, $\beta(\omega_r)$ is modified from the unmagnetized value [1] by the factors F_e and F_i, given by Eq. (12).

To summarize, we have investigated the influence of an external magnetic field on dust charge perturbations in a dusty plasma. It is found that the external magnetic field reduces the charging currents on the dust grains, thereby decreasing the dust charge fluctuation damping of a low-frequency electrostatic wave through the reduction of $\beta(\omega_r)$ by the factors F_e and F_i, which are given by Eq. (12) or Eq. (13). In order to have some appreciation of our results, we estimate the frequency and the damping rate by taking the plasma and wave parameters that are relevant for laboratory discharges. Accordingly, we take $n_{i0} \sim 10^9$ cm^{-3}, $n_{d0} \sim 10^4$ cm^{-3}, $Z_{d0} \sim 5 \times 10^3$, $T_e \sim 1$ eV, $T_i \sim 0.1$ eV, $B_0 \sim 400$ G, and $a = 10$ μm. For $k_\perp \sim 1$ cm^{-1} and $k_\parallel^2/k_\perp^2 \ll Z_{d0}^2 n_{d0} m_e/n_{e0} m_d$, the frequency and the damping rate of the dust lower hybrid are 10 s^{-1} and 2 s^{-1}, respectively. In conclusion, we state that knowledge of dust charge fluctuation induced damping of waves is required in calculating the thermal wave spectrum by using a fluctuation dissipation theorem.

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